



***Twenty-Fourth Annual
University of North Georgia
Mathematics Tournament
April 7, 2018***

Morning Component

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.

The scoring will be done by giving one point for each question answered correctly and zero points for each question answered incorrectly or left blank. Thus, it is to your advantage to answer as many questions as possible, even if you have to guess. If there is a tie, question number 8 will be used again as a tie-breaker.

This test was designed to be a CHALLENGE. It is difficult, and you may not have time to complete all questions. Do not worry if you are unable to answer several of the questions. Instead, we hope that you will obtain satisfaction from those questions which you ARE able to answer.

You may write in the test booklet. You may keep your test booklet and any of your scrap papers. Only the electronic grading form will be collected and graded.

Good luck!

Do Not Open Until Signaled.

3. If $u = f \circ g \circ h$, $f(0) = \frac{1}{2}$, $g(2) = 0$, $g'(2) = \frac{1}{2}$, $h(2) = 2$, and $h'(2) = 2$, then $u'(2) = ?$

a) $\frac{1}{2}$

b) $\frac{1}{2}$

c) 2

d) 2

e) None of the above

4. Assume that the function f is differentiable at $a \neq 0$, $f(a) \neq 0$, and $f'(a) = \frac{1}{a}$.

Find the limit: $\lim_{x \rightarrow a} \frac{x f(a) - a f(x)}{x - a}$.

a) Does not exist

b) 1

c) 0

d) 1

e) None of the above

5. The distance between the origin O and the x -intercept of the tangent line to the positive branch of the hyperbola $xy = 4$ is increasing 3 units per second. Let B be the ~~the~~

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6. The graph of the derivative of the function f , continuous on the interval $[-5, 5]$, is given. What is the correct ordering of the values $f(-5)$, $f(0)$, and

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11. Find the maximum value of the function $f(x) = \frac{\sin x - 1}{\sin^2 x - \sin x + 1}$.

- a) The maximum value of the function f is $\frac{2}{3}$.
- b) The maximum value of the function f is 0.
- c) The maximum value of the function f is 1.
- d) The maximum value of the function f does not exist.
- e) None of the above.

12. Find the limit: $\lim_{x \rightarrow 1} x \ln \left(1 + \frac{x}{2}\right) - \ln \frac{x}{2}$.

- a) Does not exist
- b) 0
- c) 1
- d) 2
- e) None of the above

13. A tangent line to the graph of $y = x^3$ at a point (x_0, y_0) has been drawn. What is (x_0, y_0) if the tangent line passes through the point $(2, 4)$ as well?

- a) $(1, 2)$
- b) $(1 - \sqrt{3}, 10)$
- c) $(1, 10 - 6\sqrt{3})$
- d) $(1 - \sqrt{3}, 8)$
- e) None of the above

14. Evaluate the following limit by first recognizing the sum as a Riemann sum for a function

defined on $[0, 1]$: $\lim_x \frac{n}{n^2} \cdot 1 + \frac{n}{n^2} \cdot 2^2 + \dots + \frac{n}{n^2} \cdot n^2 \cdot \Delta x$

a) $\frac{1}{2}$

b) $-\frac{1}{2}$

17. Car A travels west at 50 mi/h and car B travels north at 60 mi/h (See picture below). Both are heading for the intersection of the two roads. At what rate are the cars approaching each other when car A is 0.3 mi and car B is 0.4 mi from the intersection?

- a) 78 mi/h
- b) 80 mi/h
- c) 50 mi/h
- d) 67 mi/h
- e) None of the above

18. Evaluate the definite integral: $\int_0^1 \frac{(7x-1)^{2018}}{2020} dx$

19. Let f be a continuous function. Find $f(4)$ if $\int_0^{x^2} f(t) dt = x \sin x$ for all x .

- a) $\frac{1}{4}$
- b) $\frac{1}{3}$
- c) $\frac{1}{2}$
- d)
- e) None of the above

20. Find the limit: $\lim_{x \rightarrow 0} \frac{\cos \frac{x}{2} - \cos x}{\sin(\sin x)}$.

- a) Does not exist
- b) 0
- c) 1
- d) $\frac{1}{2}$
- e) None of the above

21. Find minimum value of the function $y = (19 - x)e^{19 - x}$.

- a) $\frac{19}{e^{19}}$
- b) $\frac{1}{e}$
- c) $\frac{1}{2}$

22. If $K(r) = \frac{1}{2}\sin(2r)$, find $K^{(51)}\left(\frac{\pi}{8}\right)$.

- a) $2^{50}\sqrt{2}$
- b) $2^{50}\sqrt{2}$
- c) $2^{49}\sqrt{2}$
- d) $2^{49}\sqrt{2}$
- e) None of the above

23. Find the limit: $\lim_{x \rightarrow 0} x^2 x^{\frac{1}{3}} \sin \frac{1}{x^2}$.

- a) 0
- b) 1
- c) $\frac{2}{3}$
- d) Does not exist
- e) None of the above

24. Find the integral: $\int \frac{dx}{(ax-1)(x^2-1)}$, $a > 0$.

- a) $\frac{1}{a^2-1} a \ln|ax-1| - \frac{a}{2} \ln(x^2-1) + \arctan(x) + C$
- b) $\frac{1}{a} \ln|ax-1| + \arctan(x) + C$
- c) $(a^2-1) \arctan(x) - \frac{a}{2} \ln|x^2-1| + C$
- d) $\frac{a}{2} \ln|x^2-1| + C$
- e) None of the above

25. Find the area of the surface of revolution generated by revolving $x^{\frac{2}{3}} y^{\frac{2}{3}} a^{\frac{2}{3}}$, for $a > 0$, about the x -axis.

a) $\frac{6 a^2}{5}$

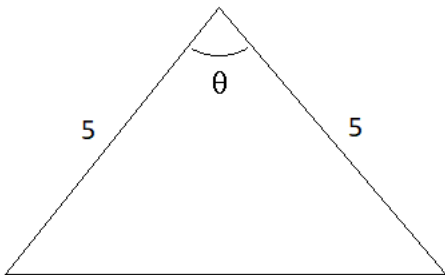
b) $\frac{12 a^2}{5}$

c) $\frac{4 a^2}{5}$

d) $\frac{8 a^2}{5}$

e) None of the above

26. What angle



27. Let $f(x) = \begin{cases} \frac{\cos x}{2} & \text{if } x < \frac{\pi}{2} \\ a^2 - a + 1 & \text{if } x = \frac{\pi}{2} \end{cases}$

For which value of a is the function f continuous at $\frac{\pi}{2}$?

- a) $a = 1$ \emptyset
- b) $a = 2$
- c) $a = 0$ and $a = 1$
- d) $a = 2$
- e) None of the above

28. Evaluate the definite integral: $\int_0^{2a} \frac{dx}{(x-a)^2}$

- a) $\frac{2}{a}$
- b) $\frac{2}{a}$

29. Let f be a continuous function defined for all real numbers having the following properties.

$$f(0) = 0$$

$$f(-1) = 0$$

$$f(x) = 0 \text{ if } 0 \leq x \leq 2$$

Which of the following could be a part of the graph of f ?

a)

b)

c)

d)

e) None of the above

30. Find a monic polynomial, P , of degree 3 such that the graph of P has a local (or relative) maximum at $(x, y) = (3, 10)$ and a point of inflection when $x = \frac{5}{3}$.

a) $P(x) = x^3 - 2x - 3$

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34. If $\tan(xy) = xy$, find $\frac{dy}{dx}$.

a) $\frac{x}{y}$

b) $\frac{x}{y}$

c) $\frac{y}{x}$

d) $\frac{y}{x}$

e) None of the above

35. Given $F(x) = x^2$ is an antiderivative of the function $f(x)e^{2x}$. Find the antiderivative of the function

