

***Twenty Second Annual University of North Georgia
Mathematics Tournament***

You may write in this test booklet. Only the electronic form will be graded. Correct answers are awarded one point. Incorrect or blank answers are awarded 0 points.

1. Find the integral: $\int_0^{\frac{\pi}{2}} \sqrt{1 - \sin x} \, dx$

- a) 4
- b) $\sqrt{1}$
- c) 2
- d)

3. Find the maximum value of $y = 6\cos x + 14x - 5$ on $[\frac{3}{2}, 0]$.

- a) 1
- b) 3
- c) 5
- d) 7
- e) None of the above

4. If g is the inverse function of $f(x) = 2x + \ln x$, find $g(2)$.

- a) $\frac{1}{3}$
- b) $2 + e^2$
- c) $\frac{3}{2}$
- d) $\frac{1}{2}$
- e) None of the above

5. Find the definite integral: $\int_0^1 \sqrt{x + x^2} dx$

6. If f is continuous, then $\int_0^1 f(1-x) dx$ is equal to

a) $\int_0^1 f(x) dx$

b) $\int_0^1 f(x) dx$

c) $\int_0^1 f(x) dx$

d) $\int_0^1 f(x) dx$

e) None of the above

7. Find $f(t)$ so that $\int f(t) dt = \frac{t}{2} \sqrt{4-t^2} + 2 \arcsin \frac{t}{2}$.

a) $\frac{1}{\sqrt{1-\frac{t^2}{4}}} C$

b) $\sqrt{4-t^2} C$

c) $\arcsin \frac{t}{2} C$

d) $t\sqrt{4-t^2} C$

e) None of the above

8. Find the number b such that the line $y = b$ divides the region bounded by the curves $y = x^2$ and $y = 4$ into two regions with equal area.

a) $4^{2/3}$

b)

9. Find the definite integral: $\int_0^1 \arcsin x \, dx$

- a) 1
- b) $\frac{1}{2}$
- c) $\frac{1}{2}$
- d) $\frac{1}{3}$
- e) None of the above

10. Suppose that $\int_0^x f(t) \, dt = \sin x$, find $f(x)$.

- a) 1
- b) -1
- c) 0
- d) \sqrt{x}
- e) None of the above

11. Evaluate the limit: $\lim_{x \rightarrow \infty} \frac{\arctan x}{\ln \frac{2x-1}{1-2x}}$.

- a)

12. A light house is located on a small island, 3 km away from the nearest point P on a straight shoreline, and its light makes four revolutions per minute. How fast is the beam light moving along the shoreline when it is 1 km from P ?

- a) 1600 km/h
- b) 480 km/h
- c) 1800 km/h
- d) 7200 km/h
- e) None of the above

13. If f is continuous and $\int_0^2 f(x) dx = 6$, evaluate $\int_0^{\frac{\pi}{2}} f(2\sin \theta) \cos \theta d\theta$.

- a) 3
- b) 6
- c) 2
- d) 12
- e) None of the above

14. Assume that f is continuous and that $f(1) = 3$, $f'(1) = 2$, and $\int_0^1 f(x) dx = 5$.

Find $\int_0^1 x^2 f'(x) dx$.

- a) 6
- b) 1
- c) $\frac{1}{2}$
- d) 2
- e) None of the above

17. Evaluate the integral: $\int \frac{dx}{1 - e^x}$

- a) $x - 1 + C$
- b) $x - \ln e^x + C$
- c) $x - \ln e^{-x} + C$
- d) $x - \ln |e^x - 1| + C$
- e) None of the above

18. Water is poured into a conical cup at the rate of $\frac{2}{3}$ cubic inches per second. If the cup is 6 inches tall and if the top of the cup has a radius of 2 inches, how fast is the water level rising when the water is 4 inches deep?

- a) $\frac{3}{8}$ in/sec
- b) $\frac{3}{8}$ in/sec
- c) $\frac{1}{4}$ in/sec
- d) in/sec
- e) None of the above

19. Which of the following expressions equals to _____ ?

- a) 0
- b) $(-1)^n \frac{n!}{x^n}$
- c) $(-1)^{n-1} \frac{(n-1)!}{x^n}$

20. Let $f(k) = \frac{d^j}{dx^j} e^{kx}$. Find $f(k)$.

a) $k^{j-1} e^{kx} k^2 j$

b) $k^{j-1} e^{kx} k j^2$

c) $k^{j-1} e^{kx} kx j$

d) $k^{j-1} e^{kx} x jk$

e) None of the above

21. Find the definite integral:

$\int_0^1 \sqrt{x} dx$

a) $\frac{6}{5}$

b) $\frac{4}{9}$

c) $\frac{3}{2}$

d) $\frac{1}{3}$

e) None of the above

22. The sequence of numbers $\frac{2}{1}, \frac{3}{2}, \frac{4}{3}, \dots, \frac{101}{100}, \dots$ gets as close as you want to:

23. What is the minimum vertical distance between the parabolas $y = x^2 - 1$ and $y = x + x^2$.

- a) $\frac{7}{8}$
- b) $\frac{5}{8}$
- c) $\frac{9}{8}$
- d) $\frac{1}{8}$
- e) None of the above

24. If the radius of the circle increases from r_1 to r_2 , the average rate of change of the area of the circle with respect to the radius is

- a) Greater than $2r_2$
- b) Less than $2r_1$
- c) Equal to $2 \frac{r_1 + r_2}{2}$
- d) Equal to
- e) None of the above

25. Find the limit: $\lim_{x \rightarrow 4} \sqrt{x^2 - 4x} - x$

- a) 1
- b) 2
- c) 4
- d) Does not exist
- e) None of the above

29. Suppose you have two linear functions f and g shown below.

$$\text{Then } \lim_{x \rightarrow a} \frac{f(x)}{g(x)}$$

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37. Find the length of the curve with the equation $x^{2/3} + y^{2/3} = 1$

- a) 6
- b) 4
- c) 24
- d) 12
- e) None of the above

38. In the figure below, find the dimension of the rectangle with maximal area in the $45^\circ-45^\circ-90^\circ$ right triangle with legs of length 1.

- a) $\sqrt{2}, \sqrt{2}$
- b) $\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{4}$
- c) $\frac{1}{2}, \frac{1}{4}$

39. Find the derivative $\frac{df}{dx}$ of the function $f(x) = x^{\sin x}$.

- a) $\sin x \cdot x^{\sin x - 1}$
- b) $\sin x \cdot x^{\cos x}$
- c) $x^{\sin x} \cos x \ln x + \frac{\sin x}{x}$
- d) $x^{\sin x} \ln \sin x$
- e) None of the above

40. Suppose $f(x)$ is differentiable everywhere and $f'(x) = 2f(x) \sin x$ for all real x .

What is the value of $f\left(\frac{\pi}{4}\right)$?

- a) 1
- b) $\frac{1}{2}$
- c) $\frac{\sqrt{2}}{2}$
- d) $\frac{\sqrt{2}}{4}$
- e) None of the above