

*University of North Georgia*  
*Sophomore Level Mathematics Tournament*  
*April 5, 2014*

**Solutions for the Afternoon Team Competition**

Round 1

Volume =  $r^2h$   $6^2 \cdot 3$   $6 \cdot 6 \cdot 3$   $6 \cdot 2 \cdot 3 \cdot 3$   $12 \cdot 9$   
The answer is 12 pieces.

Round 2

We think about the complement — people choose different numbers.  
The first person can choose any number (positive integer less than 11: from 1 to 10), then the second person would have 9 (different) numbers to choose (9/10), the third person 8 (different) numbers to choose, etc. So the probability that the 4 people choose different numbers is:

$$1 - \frac{9}{10} \cdot \frac{8}{10} \cdot \frac{7}{10} = \frac{504}{1000}$$

Hence the probability that two of the people choose the same number is:

$$1 - \frac{504}{1000} = \frac{496}{1000} = 0.496.$$

Round 3

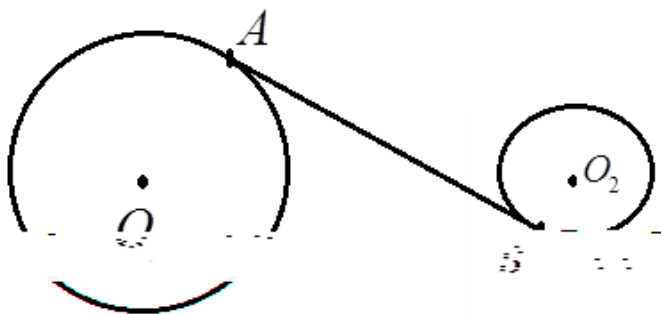
Since  $f(x)$  is divisible by  $(x-1)^3$ ,  $x^4 - ax^2 - bx - c = (x-1)^3(x-d)$  for some real number  $d$ .

Now if we equate the coefficient of  $x^3$  on both sides we see that  $d = 3$ .

Then  $f(x) = x^4 - 2x^3 + 2x^2 - 3x + 5$ .

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Round 4



We get 16 and 8 from the fact that the triangles are congruent. Then we use the Pythagorean Theorem twice getting  $a = \sqrt{220} = 2\sqrt{55}$  and  $b = \sqrt{55}$ . So  $a + b = 3\sqrt{55}$ .

Round 5

We have  $\cot \theta = \cot \phi = 4$ , so  $\frac{1}{\tan \theta} = \frac{1}{\tan \phi} = 4$  and  $\frac{\tan \theta}{\tan \phi} = 4$ .

Thus,  $\tan \theta = \tan \phi \cdot \frac{\tan \theta}{\tan \phi} = \frac{7}{4}$ .

Then

Round 6

Let  $r$  be the radius in inches. Then the area in square inches is  $r^2$  which must be a natural number according to the problem.

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Round 7

$f = 1^2 + 2^2 + 4^2 + 6^2 + \dots + 100^2$  and  $g = 1^2 + 2^2 + 4^2 + 6^2 + \dots + 98^2 + 100^2$

$g = 1^2 + 3^2 + 5^2 + \dots + 99^2 + 1^2 + 2^2 + 4^2 + 6^2 + \dots + 98^2 + 1^2 + 50^2 + 2^2 + 4^2 + 6^2 + \dots + 98^2$

The sum  $2^2 + 4^2 + 6^2 + \dots + 98^2$  can be evaluated as  $2^2(1^2 + 2^2 + 3^2 + \dots + 49^2) = 2450$ .

Consequently,  $f = 1^2 + 2450 + 100^2 = 2550$  and  $g = 1^2 + 50^2 + 2450 = 2500$ .

So  $f^2 = 2550^2$  and  $g^2 = 2500^2$ .  $f^2 - g^2 = (2550 + 2500)(2550 - 2500) = 252,500$

Dividing 252,500 by 100 gives 2525.

Round 8

We are looking for  $abcd = 1200$ , where  $a, b, c,$  and  $d$  are primes with  $a < b < c < d$ . We solve this problem by finding the largest possible value for  $a$ , then for  $b$ , and so on. It turns out you can find the answer by making a dozen or so calculations.

- |   |                                      |
|---|--------------------------------------|
|   | $2 \cdot 3 \cdot 5 \cdot 7 = 210$    |
| 1. Establish a benchmark by multiplying consecutive primes: | $3 \cdot 5 \cdot 7 \cdot 11 = 1155$  |
|   | $5 \cdot 7 \cdot 11 \cdot 13 = 5005$ |
- which is the smallest value of  $abcd$  where  $a = 3$

## Round 9

Note that paths cannot be repeated. We will count all the possible paths from  $S$  to  $F$  that pass through  $M$  or  $N$  separately and then subtract any paths that are repeated. This is known as an inclusion-exclusion method.

Part 1: Paths from  $S$  to  $F$  through  $M$  (or simply  $SMF$  paths) these go from  $S$  to  $M$  and then to  $F$ . There are exactly 3 paths from  $S$  to  $M$  (of length 3 each). There are exactly 10 paths from  $M$  to  $F$  (of length 5 each). For each of the 3  $SM$  paths, there are 10  $MF$  paths giving a total of  $3 \cdot 10 = 30$   $SMF$  paths.

Part 2: Paths from  $S$  to  $F$  through  $N$  (or simply  $SNF$  paths) these go from  $S$  to  $N$  and then to  $F$ . There are 15 paths from  $S$  to  $N$  (of length 6 each). There are 2 paths from  $N$  to  $F$  (of length 2 each). For each of the 15  $SN$  paths, there are 2  $NF$  paths giving a tot