Gainesville State College
Twelfth Annual Mathematics Tournament
April 8, 2006

Morning Component

Good morning!

Please do NOT open this booklet until given the signal to begin.

There are 40 multiple choice questions. Answer the questions on the electronic grading form by giving the best answer to each question.

4. A circular plate of radius 10 inches expands as it is heated. Use differentials to approximate the change in the area of the circle when the radius increases by 0.1 inches.

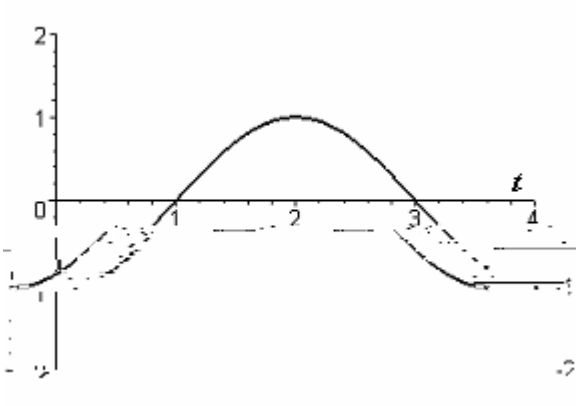
Reminder

Question 8 will be used again as a tie-breaker, if necessary.

8. A person 6 feet tall is walking away from a street light 20 feet high at a rate of 7 ft/sec. At what rate is the length of the person's shadow increasing?

a) 2 ft/sec
b) 3 ft/sec
c) 4 ft/sec
d) 5 ft/sec
e) . none of the abo?RS7':H-(tkU'1C7L9C7RR:7CC-,T d e [('e()k-LC':C7(kU-C7SL-'(3kU-L=9=-

11. The graph of the function f in the interval $[0,4]$ is given. Identify the x-coordinate of the maximum of the function g on this interval if $g(x)=\int_0^x f(t)\,dt$.



a) $x=1$
b) $x=2$
c) $x=3$
d) $x=4$
e) none of the above

12. What is the limit of the function $g(x)=2+[\![x]\!]+[\![-x]\!]$ as x approaches 2? Recall that the greatest integer function is defined by $[\![x]\!]$ = greatest integer less than or equal to x.

a) 1
b) 2
c) 3
d) 4
e) none of the above

13. Find the x- and y-intercepts of the line that is tangent to the graph of $y=x^3+x^2+x$ at the point $(-1,-1)$.

a) x-intercept is $\frac{1}{-},0$

14. Suppose that $f(1)=1$ and $f'(1)=2$. Find the value of the derivative of $f\left(f\left(f(x)\right)\right)$ at $x=1$.

a) The value of this derivative is 8.
b) The value of this derivative is 4.
c) The value of this derivative is 2.
d) The value of this derivative is 1.
e) none of the above

15. Suppose that the quadratic function $f(x)=ax^2+bx+c$ is non-negative on the interval $[-1,1]$. Then the area under the graph of f over the interval $[-1,1]$ is given by the formula

a) $A=\frac{1}{2}\left[f(-1)+2f(0)+f(1)\right]$
b) $A=f\left(-\frac{1}{2}\right)+f\left(\frac{1}{2}\right)$
c) $A=\frac{1}{3}\left[f(-1)+4f(0)+f(1)\right]$
d) $A=f(-1)+f(1)$
e) none of the above

18. What type of relative extrema does the function $f(x) = x^{\frac{2}{3}}e^{-3x}$ have?

a) one relative maximum point, no relative minimum point
b) one relative maximum point, two relative minimum points
c) two relative maxima points, no relative minimum point
d) two relative maxima points, one relative minimum point
e) none of the above

19. Evaluate: $\lim_{x \to 0^+} \frac{e^{x^2} - e^x + x}{1 - \cos(2x)}$

a) $\frac{1}{4}$
b) $\frac{1}{2}$
c) 1
d) ∞
e) none of the above

20. How many of the following derivatives are correct (on their domains)?

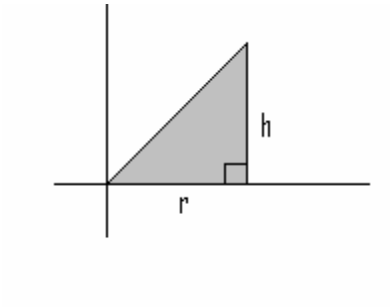
I. $\frac{d}{dx} \ln|\sec x| = \tan x$

II. $\frac{d}{dx} \ln(x + e^x) = 1 + \frac{1}{x}$

III. —

22. If $f(x)$ is differentiable at a, what is

25. If the region shown is rotated around the y-axis, the resulting volume is



a) $\frac{1}{3}\pi r^2 h$

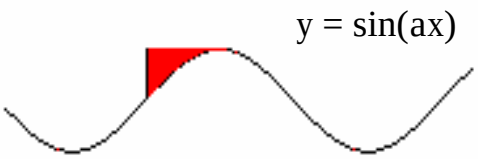
b) $\frac{2}{3}\pi r^2 h$

c) $\frac{1}{3}\pi r h^2$

d) $\frac{2}{3}\pi r h^2$

e) none of the above

26. For what value of $a > 0$ is the area of the shaded region equal to 1?



a) $\frac{\pi}{4}$

b) 1

c) $\frac{\pi}{2}$

d) $\frac{\pi}{2} - 1$

e) none of the above

27. Evaluate: $\lim_{x \to 0} \frac{2^x - 1}{x}$

a) 0

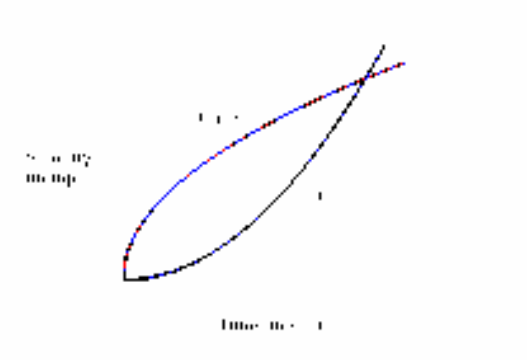
b) 1

c) $\ln(2)$

d) does not exist

e) none of the above

28. Let $[\![x]\!]$ denote the greatest integer function, that is $[\![x]\!]$ = greatest integer less than or equal to x



31. Find the derivative of $f(x) = x^e e^x$.

a) $x^{e+1}e^{x-1}$
b) $x^e e^x + x^{e+1}e^{x+1}$
c) $x^e e^x + x^{e-1}e^{x-1}$
d) $x^e e^x + x^{e-1}e^{x+1}$
e) none of the above

32. Evaluate: $\int_0^{2\pi} 3^{\sin^2 x}\, 3^{\cos^2 x}\, dx$

a) $3^{2\pi}$
b) 2π
c) 6π
d) undefined
e) none of the above

33. Let the function f be differentiable such that $f(0) = 0$

35. Evaluate: $\lim_{x \to 3^+} \frac{\ln x^3}{x-3}$

a) ∞
b) e^3
c) 0
d) 1
e) none of the above

36. When $y^3 + y = x$ then $\frac{d^2y}{dx^2}$ is equal to

a) $\frac{-6y}{\left(3x^2+1\right)^3}$
b) $\frac{-6x}{\left(3x^2+1\right)^3}$
c) $\frac{-6y}{\left(3y^2+1\right)^3}$
d) $\frac{-6x}{\left(3y^2+1\right)^3}$
e) none of the above

37. If $f(x) = g(1-x)$, and $\int_0^1 f(x)\,dx = 2$, then $\int_0^1 g(x)\,dx =$

a) -2
b) -1
c) 0
d) 2
e) none of the above

38. If p and q are positive numbers and $\int_0^p f(x)\,dx = 10$, for which of the following values of q must $\int_0^q 2x\, f(x^2)\,dx = 10$?

a) $q = p$
b) $q = p^2$
c) $q = \sqrt{p}$
d) $q = \frac{p}{2}$
e) none of the above

39. Let R be the region in the first quadrant bounded by $x = 0$, $y = 0$, and $y = 1 - x^2$.
Let A be the volume of the solid obtained by rotating this region around the x-axis.
Let B be the volume of the solid obtained by rotating this region around the y-axis.
Let C be the volume of the solid obtained by rotating this region around the line $x = 1$.

Which is largest?

a) A
b) B
c) C
d) $A = B = C$
e) none of the above

40. If $f(x)$ is differentiable and its derivative is everywhere continuous, then $\lim_{h \to 0} \frac{f(a+3h) - f(a)}{h} =$

a) $3f'(a)$
b) $f'(a)$
c) $f'(3a)$
d) $\frac{1}{3} f'(a)$
e) none of the above